

## Chapter 1: Introduction to Cognitive Modeling

Cognitive models are appearing in all fields of cognition at a rapidly increasing rate, ranging from perception to memory to problem solving and decision-making. Over 80% of the articles appearing in major theoretical journals of Cognitive Science involve cognitive modeling.<sup>1</sup> Furthermore, applications of cognitive modeling are beginning to spill over into other fields including human factors, clinical psychology, cognitive neuroscience, agent based modeling in economics, and many more.<sup>2</sup> Thus cognitive modeling is becoming an essential tool for Cognitive Science in particular and the Social Sciences in general, and any serious student with an inclination toward theory development needs to become a competent reader and perhaps user of these tools.

*A Brief Example of a Cognitive Model.* To get a quick idea about cognitive models, here is a briefly described example. One highly active area of cognitive modeling is concerned with the question of how we learn to categorize perceptual objects. For example, how does a radiologist learn to categorize whether an x-ray image contains a cancerous tumor, a benign tumor, or no tumor at all? How does a naturalist learn to categorize wild mushrooms as poisonous, edible, or harmless but inedible? How does an amateur art enthusiast learn to categorize paintings as belonging to the renaissance period, or romantic period, or modern period, or some other period?

One cognitive model is called the prototype model of categorization. According to this model, the learner estimates the central tendency from all the examples experienced

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<sup>1</sup> such as, Cognition, Cognitive Psychology, Cognitive Science, Neural Networks, Psychological Review, Psychonomic Bulletin and Review, Journal of Mathematical Psychology.

<sup>2</sup> Examples are the special issues in Psychological Assessment (2002), Human Factors (2002), Cognitive Science (2008), Journal of Mathematical Psychology (2009)

from within each category during training. When a new target stimulus is presented, the similarity of this target to each category prototype is evaluated, and the category with the most similar prototype is chosen. Another cognitive model is called the exemplar model of categorization. According to this model, the learner memorizes all the examples that are experienced from each category during training. When a new target stimulus is presented, the similarity of the target to each stored example is computed for each category, and the category with the greatest total similarity is chosen.

Prototype and exemplar models are just two examples of categorization models, and there are many more in the literature, including artificial neural network models, decision tree models, and production rule models of categorization. These models differ in terms of the assumptions they make regarding what is learned and how the category decision is made. But all of these models must try to account for a common set of empirical laws or basic facts that has accumulated from experiments on categorization.

*What are Cognitive Models?* The above examples illustrate what we view as cognitive models. But what makes these models cognitive models as opposed to some other kind of models, such as conceptual models, or statistical models or neural models? Cognitive science is concerned with understanding the processes that the brain, especially the human brain, uses to accomplish complex tasks including perceiving, learning, remembering, thinking, predicting, inference, problem solving, decision making, planning, and moving around the environment. The goal of a cognitive model is to scientifically explain one or more of these basic cognitive processes, or explain how these processes interact.

One hallmark of cognitive models is that they are described in formal, mathematical or computer, languages. Cognitive models differ from conceptual frameworks in that the latter are broadly stated, natural language (verbal) descriptions of the theoretical assumptions. For example, Craik and Lockhart's (1972) "levels of processing" hypothesis provides a conceptual framework for memory, whereas Shiffrin and Steyver's (1997) REM model or Murdock's (1993) TODAM model, being mathematical, are examples of cognitive models of memory.

Another hallmark of cognitive models is that they are derived from basic principles of cognition (Anderson & Lebiere, 1998). This is what makes cognitive models different from generic statistical models or empirical curve fitting models. For example, regression models, factor analysis models, structural equation models, and time series models are generally applicable to data from any field, as long as that data meets the statistical assumptions, such as for example, normality and linearity. These statistical assumptions are not derived from any principles of cognition, and they may even be inconsistent with the known facts of cognition. For example, the normality and homogeneity assumptions of linear regression models are inconsistent with the fact that response time distributions are positively skewed and the variance increases with the mean. These basic facts are accommodated by cognitive models of response time, such as Link and Heath's (1975) random walk model or Ratcliff's (1978) diffusion model. It is possible to use statistical tools to analyze cognitive models. For example, the parameters of random walk or diffusion models can be estimated using maximum likelihood methods, and the model can be evaluated by chi square lack of fit tests.

Cognitive models are also different from neural models, although the two can be interrelated. Cognitive models serve to build a bridge between behavior and its neural underpinnings. Cognitive models describe human information processing at a more abstract and mathematical level of analysis. Ideally we need to build bridges between the fine grain neural models and the more abstract cognitive models. To some extent, connectionist models strive to achieve this balance by building mathematical models that retain some of the properties of neural models (Rumelhart & McClelland, 1986).

*What are the advantages of Cognitive Models?* The main advantage of cognitive models over conceptual frameworks is that, by using mathematical or computer languages, cognitive models are guaranteed to produce logically valid predictions. This is not true of conclusions based on intuitively based verbal reasoning. For example, early categorization researchers argued in favor of a prototype hypothesis over an exemplar hypothesis on the basis of the fact that a prototype stimulus, which was never experienced during training, was categorized more accurately than a training stimulus, on a subsequent delayed transfer test. However, to category researcher's surprise, once the exemplar model was mathematically developed and tested, it was shown that their logic was incorrect, and the exemplar model could easily account for superior performance to the prototype stimulus. In this case, reasoning from a conceptual framework led to incorrect conclusions about evidence for or against competing theories of category learning.

A second important reason for using mathematical or computer models is that they are capable of making precise quantitative predictions. This is not possible solely on the basis of a conceptual or verbal framework. Most researchers would reject a model

whose predictions are an order of magnitude off the mark, even though the model makes the correct qualitative or ordinal prediction.<sup>3</sup> Thus it is essential to examine both the qualitative as well as quantitative predictions of a model. Of course, it is always possible to convert a conceptual model into a cognitive model by formalizing the conceptual model (recasting the verbal statements into mathematical or computer language). In fact, this is a common method for developing cognitive models.

What are the advantages of cognitive models over generic statistical models or empirical curve fitting? Both use formal language and are capable of generating quantitative predictions. The answer is generalizability. For example, Newell (1990) reviewed a large number of studies of skill learning and formulated what he called the power law of practice -- mean response time to perform a complex task decreases according to a power function of the number of training trials. The power function model provides a good quantitative fit and empirical summary of the findings, but it is not based on any cognitive principles. Logan (1988) formulated a cognitive model for the power law of practice, called the multiple-trace model. According to this model, a new memory trace is stored after each practice trial with a randomly determined retrieval time, and the observed response time is determined by the first trace to be retrieved from memory. Based on these cognitive principles, Logan mathematically derived and explained the power law of practice. The advantage of the multiple-trace model over the power function model is that the cognitive model can be used to derive new predictions for new

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<sup>3</sup> There is an interesting story by Galileo regarding his experiment of dropping a heavy and a light ball from the leaning tower of Pisa. The Jesuits, who believed in Aristotle's theory predicted that the light ball would land after the heavy ball, whereas Galileo's theory predicted no difference in arrival times. In fact, the light ball did land a very tiny split second after the heavy ball (due to wind resistance). On the basis of the qualitative result, the Jesuits claimed that they were correct. But Galileo ironically pointed out that the error in Aristotle's prediction was several orders of magnitude larger than the prediction error from Galileo's theory. Science eventually sided with Galileo.

relationships that go far beyond the original data. For example, the cognitive model can also be used to predict how the variance of the response time changes with practice, and how accuracy changes with practice, both of which happen not to follow a power law.

Finally, why not directly build neural models and skip over cognitive models? On the one hand, cognitive models provide an abstract level of analysis that makes it computationally feasible to derive precise predictions for complex tasks and multiple measures of behavior (such as choice accuracy, choice response time, and choice confidence), which is often computationally too difficult to do with fine grain neural models. On the other hand, neural models (e.g. Grossberg, 1982; O'Reilly & Munakata, 2000) describe the actual neural substrates and neural interconnections that implement these cognitive processes, and so they are better for inferring predictions for bold activation patterns from fMRI images or neural activation patterns from multiple cell recording studies. However, the fine grain level of analysis required to build neural models (involving possibly thousands of neural interconnections) generally make them too difficult to scale up to address complex cognitive tasks. In sum, both types of models serve an important but somewhat different goals (with respect to measures that they try to predict), and so both are needed along with bridges that relate the two types of models (cf. Marr, 1982).

*What are the Practical Uses of Cognitive Models?* There are many practical uses for cognitive models in a wide variety of areas. Clinical psychologists use cognitive models to assess individual differences in cognitive processing between normal individuals and clinical patients (e.g., schizophrenics). Cognitive neuroscientists use cognitive models to understand the psychological function of different brain regions.

Aging researchers use cognitive models to understand the aging process and the deterioration or slow down of cognitive functioning with age. Human factors researchers use cognitive models to improve human - machine or human - computer interactions. Decision researchers use cognitive models of decision making to predict preferences for consumer products, investment portfolios for businesses, medical choices, and military strategies. Artificial intelligence and robotic researchers use cognitive models for automated detection of dangerous targets, automated recognition of handwriting, automated recognition of faces, or approach and avoidance movement behavior of robots. Researchers in social science such as economics and sociology use cognitive models to construct computerized agents in agent based models of market behavior or social network working.

*What are the Steps of Cognitive Modeling?* The first step is to take a conceptual theoretical framework, and reformulate its assumptions into a more rigorous mathematical or computer language description. Consider, for example, the prototype model that was briefly sketched earlier. We would need to formulate the prototype for each category as a vector of features, and write down formulas describing how to compute the distance between the target stimulus vectors and the prototype vectors. This first step uses the basic cognitive principles of the conceptual theory to construct the model.

Often the conceptual theory is insufficient or too weak to completely specify a model, or it is missing important details. In this case, the second step is to make additional detailed assumptions (called ad hoc assumptions) in order to complete the model. This is often necessary for generating precise quantitative predictions. For

example, considering the prototype model, we would need to make detailed, but somewhat ad hoc, assumptions about what features should be used to represent the stimuli to be categorized. Theorists try to minimize the number of ad hoc assumptions, but this step is often unavoidable.

Models almost always contain parameters, or coefficients that are initially unknown, and the third step is to estimate these parameters from some of the observed data. For example, the prototype model may include weight parameters that determine the importance of each feature for the categorization problem. The importance weight assigned to each feature is a free parameter that is estimated from the choice response data (analogous to the problem of estimating regression coefficients in a linear regression model). Theorists try to minimize the number of model parameters, but this is usually a necessary and important step of modeling.

The fourth step is to compare the predictions of competing models with respect to their ability to explain the empirical results. It is meaningless to ask if a model can fit the data or not (Roberts & Pashler, 2000). In fact, all models are deliberately constructed to be simple representations that only capture the essentials of the cognitive systems. Thus we know, a priori, that all models are wrong in some details, and a sufficient amount of data will always prove that a model is not true. The question we need to ask is -- which model provides a better representation of the cognitive system that we are trying to represent? For example, we know from the beginning that both the prototype and the exemplar models are wrong in detail, but we want to know which of these two models provide a better explanation of how we categorize objects.

To empirically test competing models, researchers try to design experimental conditions that lead to opposite qualitative or ordinal predictions from the two models (e.g., the prototype model predicts that stimulus X is categorized in category A most often, but the exemplar model predicts that stimulus X is categorized in category B most often). These qualitative tests are designed to be parameter free in the sense that the models are forced to make these predictions for any value of the free parameters. However, it is not always possible to construct qualitative tests for deciding between the models, and we often need to resort to quantitative tests in which we compare the magnitude of the prediction errors produced by each model. Even if it is possible to construct qualitative tests, it is also informative to examine the quantitative accuracy as well.

The last step is often to start all over and reformulate the theoretical framework and construct new models in light of the feedback obtained from new experimental results. Model development and testing is a never-ending process. New experimental findings are discovered all the time, posing new challenges to previous models. Previous models need to be modified or extended to account for these new results, or in some cases, we need to discard the old models and start all over. Thus the modeling process produces an evolution of models that improve and become more powerful over time as the science in a field progresses.